Journal homepage www.jzs.univsul.edu.iq
Journal of Zankoy Sulaimani
Part-A- (Pure and Applied Sciences)

# A Proposed Method to Solve Quadratic Fractional Programming Problem by Converting to Double Linear Programming

Amir Sabir Majeed[1] & Fadhil Salman Abed[2]

*1 Sulaimani Polytechnic University, of Kalar Technical Institute, Iraq E-Mail:* amirsabr@yahoo.com
*2 Sulaimani Polytechnic University, of Kalar Technical Institute,Iraq, E-Mail:* fad_Sal_Abed@Yahoo.com

**Article info**

Original : 7 Spet. 2015
Revised: 23 May 2016
Accepted: 5 June 2016
Published online: 20 March 2017

***Key Words:***
QFPP, LP, simplex method, Optimality Conditions, quadratic programming, fractional programming, linearization.

**Abstract**

Quadratic fractional program is an optimization problem which solving the problem by minimizes or maximizes a quadratic fractional objective function subject to finite number of linear inequality (equality constraints) . In our paper, we proposed a new method to solve quadratic fractional programming problem (QFPP),the objective function of quadratic optimization has linear factorized as product of two linear functions, the two positive linear functions solved separately by using simplex method. These are useful in solving the problem in multi-application, like economics, hospital and health, engineering problem, financial planning etc.
In our paper, it was addressed to a variety of examples and the results were encouraging and accurate comparison with other methods with ease in the solution.

## 1. Introduction

The problems of QFPP are very importance subjects in nonlinear programming ,Like linear programming, nonlinear programming is a mathematical technique for determining the optimal solution to many business problems, The non-linear fractional programming problem, i.e. the (minimization) maximization of a fraction of two separated functions subject to given constricted conditions, applied in various decision making problems; for example linear fractional programming is used in fields of network flows, transportation problems ; the QFPP is used on field production planning and inventories[2] . There are different applications of nonlinear programming, one of them encounters the problem in which the ratio of given two functions is to be minimized or maximized [1].

There are different solution method for determining the best solution of particular problems of fractional programming problems. One by Charnes and Cooper (1962), other by Isbell and Marlow (1962) ,Martos (1964) and Wolf (1985) solved linear fractional programming problem.[2]. Bitran and Novaes [9] solve the LFP by solving a sequence of linear programs only re-computing the local gradient of the objective function. Dinkelbach, 1967, used his algorithm for convex nonlinear fractional programming problems.[3] i.e Several methods to solve such problems are proposed in (1962), their method depends on transforming this LFPP to an equivalent Linear Program[4] , Rajendra (1993) solved the Integer linear fractional programming. Gopal et al.(1991) went to investigates configuration management and optimal logical network design for reconfigurable networks. They defined underlying constrained non-linear integer fractional problems and developed a heuristic technique to solve it. . Archana Khurana and Arora (2011) solving the problem of QFP under some of its constraints are homogeneous . M. Biggs at (2005) solving nonlinear Optimization [5].

The special case problem will be solved by simplex method after convert the objective function to double linear fractional programs and after word converting each linear fractional to linear programming

**Theorem:** Assume that a LP in standard form is feasible and the optimal objective value is finite. There exists an optimal solution which is an extreme point[6].

- Simplex Method
  Which is very efficient in practice but specialized for LP: move from one vertex to another without enumerating all the vertices
- Interior-point Method [7].
- Ellipsoid Method
  This long standing issue was resolved by Leonid Khachiyan in 1979 with the introduction of the ellipsoid method, the first worst-case polynomial-time algorithm for linear programming.
- Cutting-plane Method[8].

Section below illustrated the relationship between LP and QFP

The mathematical form of an LP is as follows:

Maximize(minimize) $Z = c\,x$ (1)

Subject to $A\,x = b$ (2)

$x \geq 0$ (3)

$b \geq 0$ (4)

where $A = (a_1, a_2, \ldots, a_m, a_{m+1}, \ldots, a_n)$ is m×n matrix, $b \in R^m$, $x, c, \in R^n$, $x$ is a (n×1)column vector, and c is a (1×n) row vector,

## 3. Mathematical form of QFPP

The mathematical form of QFPP is given as follows:

$$\text{Max } z = \frac{(C^T x + \delta + \frac{1}{2} x^T G x)}{(d^T x + \gamma)} \quad (5)$$

Subject to

$$Ax \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} b, \quad (6)$$

$$x \geq 0 \quad (7)$$

Where **G** is **(n× m)** matrix of coefficients with **G** is symmetric matrix. All vectors are assumed to be column vectors unless transposed ***(T)***, Where $x$ is an n-dimensional column vector of decision variables, C is the n-dimensional row vector of constants, b is n-dimensional column vectors vector of constants. $\gamma$ , $\delta$ are scalars.

In our paper the problem that has objective function is solved by using the following form:

$$\text{Max } z = \frac{(c_1{}^T x + \alpha)\ (c_2{}^T x + \beta)}{(d^T x + \gamma)} \quad (8)$$

Subject to

$$Ax \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} b, \quad (9)$$

$$x \geq 0 \quad (10)$$

A is m x n matrix , all vectors are assumed to be column vectors unless transposed ***(T)***. Where $x$ is an n-dimensional column vector of decision variables, $c_1$, $c_2$, d are the n-dimensional row vector of constants, **b is n**-dimensional column vector of constants, $\alpha$ , $\beta$ , $\gamma$ are scalars. Now we divide the QFP into two parts one of them is LFP of the form

$$\text{Max } z_1 = \frac{(c_1{}^T x + \alpha)}{(d^T x + \gamma)} \tag{11}$$

Subject to:

$$A\,x \begin{pmatrix} \geq \\ \leq & b \\ = \end{pmatrix} \tag{12}$$

$$x \geq 0 \tag{13}$$

and the other is LP of the form

$$\textbf{Max } \mathbf{z_2} = (\boldsymbol{c_2}^{\boldsymbol{T}} \boldsymbol{x} + \boldsymbol{\beta}) \tag{14}$$

Subject to

$$Ax = \begin{bmatrix} \geq \\ \leq \\ = \end{bmatrix} b \tag{15}$$

$$x \geq 0 \tag{16}$$

Now about equation (11)
It is assumed that the feasible region $S = \{x\epsilon R^n : Ax \leq b, x \geq 0\}$
is nonempty and bounded and the denominator $d^t x + \gamma \neq 0$

**Case 1**: if d=0 and $\gamma = 1$ then the LFP in (11) to (13) becomes an LP problem. Then (11) can be written as:

Maximize $Z_1 = (c_1{}^T x + \alpha)$ (17)

Subject to $A\,x \leq b$ ; $x \geq 0$ (18)

**Case 2:** if d=0 and $\gamma \neq 1$ in (11) , then $z_1$ becomes a linear function
$z_1 = \frac{c_1}{\gamma} x + \frac{\alpha}{\gamma} = \frac{z'}{\gamma}$ , where z' = $(c_1{}^T x + \alpha)$ is a linear function.
In this case $Z_1$ may be substituted with $\frac{z'}{\gamma}$ corresponding to the same set of feasible region S. As a result the LFP becomes an LP
**Case 3** : if $c_1$=0 in (11) then $z_1 = \frac{\alpha}{(d^T x + \gamma)} = \frac{\alpha}{z''}$ , where z" = $(d^T x + \gamma)$ is a linear function., and z" = $\frac{\alpha}{z_1}$
In this case $Z_1$ becomes linear on the same set of feasible solution S. therefore the LFP becomes an LP with the same feasible region S.
**Case 4:** if $C_1 = (c_1, c_2, c_3, \ldots\ldots, c_n)$ , $d = (d_1, d_2, d_3, \ldots\ldots, d_n)$ are linear dependent , there exists $\rho \neq 0$ such that $c_1 = \rho\, d$ then

$$z_1 = \frac{((\rho\, d)^T x + \alpha)}{dx + \gamma} = \frac{((\rho\, d)^T x + \rho\gamma - \rho\gamma + \alpha)}{(d^T x + \gamma)} = \frac{((\rho\, d)^T x + \rho\gamma}{(d^T x + \gamma)} + \frac{-\rho\gamma + \alpha)}{(d^T x + \gamma)}$$

$$z_1 = \frac{\rho(d^T x + \gamma)}{(d^T x + \gamma)} + \frac{\alpha - \rho\gamma}{(d^T x + \gamma)} \quad \text{then } z_1 = \rho + \frac{\alpha - \rho\gamma}{(d^T x + \gamma)}$$

i) If $\alpha - \rho\gamma = 0$ then $z_1 = \rho$ *is a constant.*
ii) If $\alpha - \rho\gamma \neq 0$ then Z1 becomes a linear functions. Therefore the LFP becomes LP with the same feasible region S.

If $c_1 \neq 0, d \neq 0$ then one has to find a new way to convert the LFP into an LP . Assuming that the feasible region $S=\{ x \in R^n: Ax \leq b, x \geq 0\}$
Is nonempty and bounded and the denominator $c^T x + \gamma > 0$ , we develop a method which converts an LFP of this type to a LP

**4. Proposed Method for Solving QFPP**

In this section , we will use the developed a sophisticated method for solving LFP problems to solve the QFPP.

For this , we assume that the feasible region

$$S=\{ x \in R^n: Ax \leq b, x \geq 0\}$$

Is nonempty and bounded and the denominator $c^T x + \gamma > 0$ .

In this case we will divide the quadratic fractional programming problem(QFPP) in to:

1) a linear programming (LP) factor

Max $z_2 = (c_2{}^T x + \beta)$

Subject to

$$Ax = \begin{bmatrix} \geq \\ \leq \\ = \end{bmatrix} b$$

$$x \geq 0$$

then solve the above LP in a suitable method for $x$ and let the optimal solution be $q_1$

2) linear fractional programming (LFP) factor and then we convert the (LFP)

Max $z_1 = \frac{(c_1{}^T x + \alpha)}{(d^T x + \gamma)}$

Subject to

$$Ax = \begin{bmatrix} \geq \\ \leq \\ = \end{bmatrix} b$$

$$x \geq 0$$

to (LP) for y.

We will derive the converting

i) Transformation of the objective function Max $z_1 = \frac{(c_1{}^T x + \alpha)}{(d^T x + \gamma)}$

Multiplying both the denominator and the numerator by $\gamma$ we have

$$z_1 = \frac{c_1{}^T x\gamma + \alpha\gamma}{\gamma(d^T x + \gamma)} = \frac{c_1{}^T x\gamma - d^T x\alpha + d^T x\alpha + \alpha\gamma}{\gamma(d^T x + \gamma)} = \frac{(c_1{}^T\gamma - d^T\alpha)x + (d^T x + \gamma)\alpha}{\gamma(d^T x + \gamma)}$$

$$= \frac{(c_1{}^T\gamma - d^T\alpha)x}{\gamma(d^T x + \gamma)} + \frac{(d^T x + \gamma)\alpha}{\gamma(d^T x + \gamma)} = (c_1{}^T - d^T \frac{\alpha}{\gamma}) \frac{x}{(d^T x + \gamma)} + \frac{\alpha}{\gamma} = py + g$$

Where $p = (c_1{}^T - d^T \frac{\alpha}{\gamma})$ , $y = \frac{x}{(d^T x + \gamma)}$ and $g = \frac{\alpha}{\gamma}$

Then $F(y) = py + g$

ii) Transformation of the constraint $A x \leq b$

$\frac{\gamma(Ax-b)}{\gamma(d^T x+\gamma)} \leq 0$ where $\gamma(d^T x+\gamma) > 0, if\ \gamma(d^T x+\gamma) < 0$

the conditioin $\frac{\gamma(Ax-b)}{\gamma(d^T x+\gamma)} \leq 0$ will not hold.

As a result solution to the LFP cannot be found

$$\frac{(Ax\gamma-b\gamma)}{\gamma(d^T x+\gamma)} \leq 0 \Longrightarrow \frac{(Ax\gamma-bd^T x+bd^T x-b\gamma)}{\gamma(d^T x+\gamma)} \leq 0$$

$$\frac{\gamma(A+\frac{b}{\gamma}d^T)x}{\gamma(d^T x+\gamma)} - \frac{b(d^T x+\gamma)}{\gamma(d^T x+\gamma)} \leq 0$$

$$(A+\frac{b}{\gamma}d^T)\frac{x}{(d^T x+\gamma)} - \frac{b}{\gamma} \leq 0$$

$$(A+\frac{b}{\gamma}d^T)\frac{x}{(d^T x+\gamma)} \leq \frac{b}{\gamma}$$

$Gy \leq h$ where $G=(A+\frac{b}{\gamma}d^T)$ , $h=\frac{b}{\gamma}$ and $y = \frac{x}{d^t x+\gamma}$

iii) Calculation of the unknown $x$ from

$$y = \frac{x}{(d^T x+\gamma)} \Longrightarrow x = y\,(d^T x+\gamma) \Longrightarrow x = y\,d^T x + y\gamma)$$

$$x - y\,d^T x = y\gamma \Longrightarrow x(1-y\,d^T) = y\gamma)$$

$x = \frac{y\gamma}{(1-y\,d^T)}$ which is the optimal solution and let it be $q_2$

3) Then Max z = max (z ($q_1$) ,z ($q_2$))

**Numerical examples:**

**Test 1:**

Max z = $(8x_1^2 + 24x_1 x_2 + 18x_2^2 - 2)/(6x_1 + 9x_2 + 3)$

Subject

$x_1 + 3x_2 \leq 5$

$2x_1 + x_2 \leq 2$

$x_1 , x_2 \geq 0$

**Solution:**

Max z= $\frac{(4x_1+6x_2-2)(2x_1+3x_2+1)}{6x_1+9x_2+3} = \frac{(4x_1+6x_2-2)}{6x_1+9x_2+3} \times (2x_1+3x_2+1) = z_1 . z_2$

Subject to

$x_1 + 3x_2 \leq 5$

$2x_1 + x_2 \leq 2$

$x_1 , x_2 \geq 0$

Where $z_1 = \frac{(4x_1+6x_2-2)}{6x_1+9x_2+3}$ , $z_2 = (2x_1+3x_2+1)$

Now we solve the LFP

Max $z_1 = z_1 = \frac{(4x_1+6x_2-2)}{6x_1+9x_2+3}$

Subject to

$x_1 + 3x_2 \leq 5$

$2x_1 + x_2 \leq 2$

$x_1 , x_2 \geq 0$

By convert it to LP

We have $c_1 = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $d = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$, $\alpha = -2$, $\gamma = 3$

$$p = \left( C_1^T - d^T \frac{\alpha}{\gamma} \right) = (4 \quad 6) - (6 \quad 9)\begin{pmatrix} \frac{5}{3} \end{pmatrix} = (8 \quad 12) \text{ and } g = \frac{\alpha}{\gamma} = \frac{-2}{3}$$

$$G_1 = \left( A_1 + \frac{b_1}{\gamma} d^T \right) = (1 \quad 3) + \begin{pmatrix} \frac{5}{3} \end{pmatrix}(6 \quad 9) = (11 \quad 18) \text{ and } h_1 = \frac{b_1}{\gamma} = \frac{5}{3}$$

$$G_1 = \left( A_2 + \frac{b_2}{\gamma} d^T \right) = (2 \quad 1) + \begin{pmatrix} \frac{5}{3} \end{pmatrix}(6 \quad 9) = (6 \quad 7) \text{ and } h_2 = \frac{b_2}{\gamma} = \frac{2}{3}$$

**Then**

Max $F(y)=8y_1+12y_2 - \frac{2}{3}$

Subject to

$$11y_1+18y_2 \leq \frac{5}{3}$$
$$6y_1+7y_2 \leq \frac{2}{3}$$
$$y_1, y_2 \geq 0$$

Now we will solve the above LP by simplex method.
Converting the LP in standard form we have ,

Max $F(y)=8y_1+12y_2 - \frac{2}{3}$

Subject to

$$11y_1+18y_2 + s_1 = \frac{5}{3}$$
$$6y_1+7y_2 + s_2 = \frac{2}{3}$$
$$y_1, y_2, s_1, s_2 \geq 0$$

Where $y_1$ and $y_2$ are decision variables , and $s_1$ *and* $s_2$ are slack variables
Now we get the following simplex table

**Table 1:**

| | | 8 | 12 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|
| | | **$y_1$** | **$y_2$** | **$s_1$** | **$s_2$** | **h** | **Ratio** |
| **0** | **$s_1$** | **11** | **18** | **1** | **0** | **$\frac{5}{3}$** | **$\frac{5}{24}$** ← |
| **0** | **$s_2$** | **6** | **7** | **0** | **1** | **$\frac{2}{3}$** | **$\frac{2}{21}$** |
| **$C_J$-$Z_J$** | | **8** | **12** ↑ | **0** | **0** | | |

**Table 2:**

| | | 8 | 12 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|
| | | **$y_1$** | **$y_2$** | **$s_1$** | **$s_2$** | **h** | **Ratio** |
| **12** | **$y_2$** | $\frac{11}{18}$ | 1 | $\frac{1}{18}$ | 0 | $\frac{5}{54}$ | $\frac{5}{33}$ ← |
| **0** | **$s_2$** | $\frac{31}{18}$ | 0 | $\frac{-7}{18}$ | 1 | $\frac{1}{54}$ | $\frac{1}{93}$ |
| **$C_J$-$Z_J$** | | $\frac{2}{3}$ ↑ | 0 | $\frac{-2}{3}$ | 0 | | |

**Optimal table :**

| | | 8 | 12 | 0 | 0 | |
|---|---|---|---|---|---|---|
| | | **$y_1$** | **$y_2$** | **$s_1$** | **$s_2$** | **h** |
| **12** | **$y_2$** | 0 | 1 | $\frac{95}{324}$ | $\frac{11}{3}$ | $\frac{12}{33}$ |
| **8** | **$y_1$** | 1 | 0 | | | $\frac{1}{93}$ |
| **$C_J$-$Z_J$** | | 0 ↑ | 0 | – | - | |

**Then the table of results as below:**

| | |
|---|---|
| **$y_1=\frac{1}{93}$** | **$y_2=\frac{12}{31}$** |
| $x = \frac{y\gamma}{(1 - \mathrm{y}\,\mathrm{c}^T)}$ | $(x_1, x_2) = \frac{(y_1,y_2)\gamma}{(1-\mathrm{c}^T(y_1,y_2))}$ |
| $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{\begin{bmatrix} \frac{1}{93} \\ \frac{12}{33} \end{bmatrix} x_3}{(1 - (6 \quad 9) \begin{bmatrix} \frac{1}{93} \\ \frac{12}{33} \end{bmatrix}} = \begin{bmatrix} \frac{1}{5} \\ \frac{8}{5} \end{bmatrix}$ | **Then**<br>**$x_1$= .2**<br>**$x_2$= 1.6** |

After solving the LP

$$\text{Max } z_2 = (2x_1 + 3x_2 + 1)$$

Subject to

$$x_1 + 3x_2 \le 5$$
$$2x_1 + x_2 \le 2$$
$$x_1 , x_2 \ge 0$$

We will find $x_1$=.2 and $x_2$=1.6

In this case the optimal solution of $z_1$ and $z_2$ are same point i.e $q_1$=$q_2$

Then max z=3.2

**Test 2:**

Max z = $(4x_1^2 + 12x_1 x_2 + 8x_2^2 + 4x_1 + 4x_2)/(4x_1 + 8x_2 + 4)$

Subject to

$$-2x_1 + x_2 \le 3$$
$$4x_1 + 2x_2 \le 8$$
$$x_1, x_2 \ge 0$$

## Solution:

$$Max\ z = \frac{(2x_1 + 2x_2)(2x_1 + 4x_2 + 2)}{(4x_1 + 8x_2 + 4)} = \frac{(2x_1 + 2x_2)}{(4x_1 + 8x_2 + 4)} \times (2x_1 + 4x_2 + 2) = z_1 z_2$$

Subject

$$-2x_1 + x_2 \le 3$$
$$4x_1 + 2x_2 \le 8$$
$$x_1, x_2 \ge 0$$

Where $z_1 = \frac{(2x_1 + 2x_2)}{(4x_1 + 8x_2 + 4)}$, $z_2 = (2x_1 + 4x_2 + 2)$

Now we solve the LFP

$$Max\ z_1 = \frac{(2x_1 + 2x_2)}{(4x_1 + 8x_2 + 4)}$$

Subject

$$-2x_1 + x_2 \le 3$$
$$4x_1 + 2x_2 \le 8$$
$$x_1, x_2 \ge 0$$

We have $c_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $d = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$, $\alpha = 0$, $\gamma = 4$

$p = \left( C_1^T - d^T \frac{\alpha}{\gamma} \right) = (2 \quad 2) - (4 \quad 8)\begin{pmatrix} \frac{0}{4} \end{pmatrix}, = (0 \quad 2)$ and $g = \frac{\alpha}{\gamma} = \frac{0}{4} = 0$

$G_1 = \left( A_1 + \frac{b_1}{\gamma} d^T \right) = (-2 \quad 1) + \left(\frac{3}{4}\right)(4 \quad 8) = (1 \quad 7)$ and $h_1 = \frac{b_1}{\gamma} = \frac{3}{4}$

$G_1 = \left(A_2 + \frac{b_2}{\gamma} d^T\right) = (4 \quad 2) + \left(\frac{8}{4}\right)(4 \quad 8) = (12 \quad 18)$ and $h_2 = \frac{b_2}{\gamma} = \frac{8}{4} = 2$

**Then**

Max $F(y)=2y_1+2y_2$

Subject to

$$y_1+7y_2 \leq \frac{3}{4}$$
$$12y_1+18y_2 \leq 2$$
$$y_1, y_2 \geq 0$$

Now we will solve the above LP by simplex method.

Converting the LP in standard form we have ,

Max $F(y)=2y_1+2y_2$

Subject to

$$y_1+7y_2+s_1 = \frac{3}{4}$$
$$12y_1+18y_2+s_2= 2$$
$$y_1, y_2, s_1, s_2 \geq 0$$

Where $y_1$ and $y_2$ are decision variables , and $s_1$ *and* $s_2$ are slack variables

**Table 1:**

| | | 2 | 2 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|
| | | $y_1$ | $y_2$ | $s_1$ | $s_2$ | h | Ratio |
| 0 | $s_1$ | 1 | 7 | 1 | 0 | $\frac{3}{4}$ | $\frac{3}{28}$ ← |
| 0 | $s_2$ | 12 | 18 | 0 | 1 | 2 | $\frac{2}{18}$ |
| $c_j$-$z_j$ | | 2 | 2 ↑ | 0 | 0 | | |

**Table 2:**

| | | 2 | 2 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|
| | | $y_1$ | $y_2$ | $s_1$ | $s_2$ | h | Ratio |
| 2 | $y_2$ | $\frac{1}{7}$ | 1 | $\frac{1}{7}$ | 0 | $\frac{3}{28}$ | $\frac{21}{28}$ ← |
| 0 | $s_2$ | $\frac{66}{7}$ | 0 | $\frac{-18}{7}$ | 1 | $\frac{1}{14}$ | $\frac{1}{32}$ |
| $c_j$-$z_j$ | | $\frac{12}{7}$ ↑ | 0 | $\frac{-2}{7}$ | 0 | | |

**Optimal table :**

| | | 8 | 12 | 0 | 0 | |
|---|---|---|---|---|---|---|
| | | $y_1$ | $y_2$ | $s_1$ | $s_2$ | h |
| 2 | $y_2$ | 0 | 1 | $\frac{25}{49}$ | $\frac{-1}{7}$ | $\frac{5}{54}$ |
| 2 | $y_1$ | 1 | 0 | $\frac{-3}{11}$ | $\frac{7}{66}$ | $\frac{1}{132}$ |
| $c_j-z_j$ | | 0 ↑ | 0 | − | - | |

**Then the table of results as below:**

| | |
|---|---|
| $y_1=\frac{1}{132}$ | $y_2=\frac{7}{66}$ |
| $x=\frac{y\gamma}{(1-\text{y}\,\text{c}^T)}$ | $(x_1,x_2)=\frac{(y_1,y_2)\gamma}{(1-\text{c}^T((y_1,y_2))}$ |
| $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{\begin{bmatrix} \frac{1}{32} \\ \frac{7}{66} \end{bmatrix} \times 4}{(1-(4 \quad 8)\begin{bmatrix} \frac{1}{132} \\ \frac{7}{66} \end{bmatrix}} = \begin{bmatrix} \frac{1}{4} \\ \frac{7}{2} \end{bmatrix}$ | **Then**<br>**$x_1$=.25**<br>**$x_2$= 3.5** |
| After solving the LP<br>Max $z_2=(2x_1+3x2+1)$<br>Subject to<br>$x_1+3x_2 \leq 5$<br>$2x_1+x_2 \leq 2$<br>$x_1, x_2 \leq 0$<br>We will find $x_1$=.25 and $x_2$=3.5<br>In this case the optimal solution of $z_1$ anad $z_2$ are same point i.e $q_1$=$q_2$<br>Then max z=3.75 | |

The table blow illustrated the comparison result between the three method Wolfe's, modified simplex, proposed method, we obtained the same result:

**Table 1: Comparison of the numerical results**

| **Example** | **Wolfe's, modified** | **Modified Simplex Method** | **Proposed Method** |
|---|---|---|---|
| **EX.1** | $x_1=0.2, x_2=1.6$<br>**Max z=3.2** | $x_1=0.2, x_2=1.6$<br>**Max z=3.2** | $x_1=0.2, x_{12}=1.6$<br>**Max z=3.2** |
| **EX.2** | $x_1=0.25, x_2=3.5$<br>**Max z=3.75** | $x_1=0.25, x_2=3.5$<br>**Max z=3.75** | $x_1=0.25, x_2=3.5$<br>**Max z=3.75** |

## 5. Conclusion

This paper presents a new approach to solve Fractional Programming Problems (FPPs) based split objective function into two linear programming and solved separated two linear programming by finding maximum value of QFPP . A better accuracy was remarkably observed in the solution results.

The comparisons of these methods are based on the value of objective function , the study the max z resulted of other methods are same as illustrated in table 1.

## References

[1] B. Abdulrahim, *"Solving Quadratic Programming Problem with ExtremePoints"*, M.Sc. Thesis, University of Sulaimani, Sulaimani/Iraq. (2011).

[2] Bazaraa, S.H.D. Sherali and C.M. Shetty ,*"Nonlinear Programming: Theory and Algorithms*",(2nd ed.), John Wiley & Sons Inc., New York, (1993).

[3] Dinkelbach, W.,*"On Nonlinear Fractional Programming",* Management Science, Vol. 13, pp. 492-498, (1967).

[4] Archana kuruna and. S. R. arora, *"An Algorithm for Solving Quadratic Fractional Program with Linear Homogeneous Constraints"*, Vietnam Journal of Mathematics Vol. 39. No. 4. pp. 391-404. (2011).

[5] M.Bigges,*"Nonlinear Optimization with Finantial Applications",* Kluwer Academic Publishers, England. (2005).

[6] Jeffrey Wheeler , *"Mathematical Models for Consultants, Linear Programming*", Carngie Mellon University, Math 70-460. (2009).

[7] Robert Robere, *"Interior Point Methods and Linear Programming",* University of Toronto December 13, (2012).

[8] Nicolas W. Sawaya, Ignacio E. Grossmann, *"A cutting plane method for solving linear generalized disjunctive programming problems",* Computers and Chemical Engineering Vol. 29, pp. 1891–1913. (2005).

[9] Bitran, G ,R. and Novaes, A.G., "*Linear programming with a Fractional Objective Function*", University of Soa Paulo. Brazil. (1972).